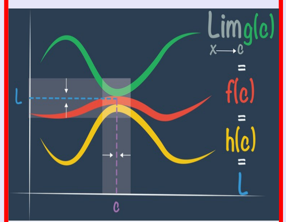
Math 261

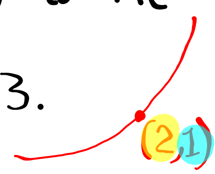
Spring 2022

Lecture 14



Class QZ 9

Find eqn of tan. line at $(2,1)$ to the curve given by $x^2+4xy+y^2=13$.



$\boxed{\text{Final Ans. in Slope-Int. Form.}}$

Verify $(2,1)$:

$2^2+4(2)(1)+1^2=13$ ✓

$2x+4\left[1\cdot y+x\cdot\frac{dy}{dx}\right]+2y\frac{dy}{dx}=0$

$2(2)+4(1+2m)+2m=0$

$4+4+8m+2m=0$

$10m=-8$

$m=\frac{-8}{10}$

$\boxed{m=\frac{-4}{5}}$

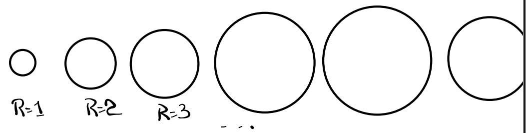
$y-1=\frac{-4}{5}(x-2)$

$y=\frac{-4}{5}x+\frac{8}{5}+1$

$\boxed{y=\frac{-4}{5}x+\frac{13}{5}}$ ✓

Consider area of a Circle $A = \pi R^2$

As R changes → A changes



As R changes with respect to time, then
A " " " " " aswell.

Take derivative of both sides of $A = \pi R^2$
with respect to time

$$\frac{d}{dt}[A] = \frac{d}{dt}[\pi R^2]$$

$$\frac{dA}{dt} = \pi \frac{d}{dt}[R^2]$$

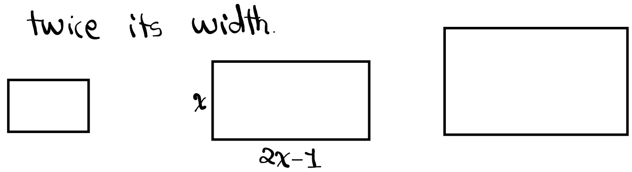
$$\frac{dA}{dt} = \pi \cdot 2R \cdot \frac{dR}{dt}$$

Suppose $\frac{dR}{dt} = 1 \text{ cm/min}$

at the time when $R = 2\text{cm}$

$$\frac{dA}{dt} = \pi \cdot 2(2) \cdot 1 \text{ cm}^2/\text{min} \Rightarrow \frac{dA}{dt} = 4\pi \text{ cm}^2/\text{min}$$

Consider a rectangle such that
its length is 1 cm shorter than
twice its width.



Area = LW
$= (2x-1)(x)$

$A(x) = 2x^2 - x$

How fast its area changes when the width is 3cm and the changes in width is 2.5 cm/min.?

$$\frac{d}{dt}[A] = \frac{d}{dt}[2x^2 - x]$$

$$\frac{dA}{dt} = (4x-1)\frac{dx}{dt}$$

$$\frac{dA}{dt} = (4\cdot 3 - 1)\cdot(2.5)$$

$$= 11 \cdot \frac{5}{2} = \frac{55}{2} \text{ cm}^2/\text{min.}$$

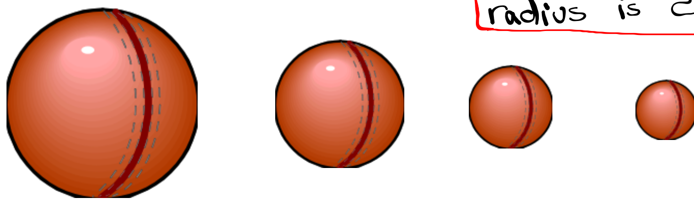
March 28, 2022

A ball is leaking air

its volume is changing at 5 in^3/hr.

$\frac{dV}{dt} = -5$

How fast is its radius changing when radius is 2 inches?



$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{\cancel{3}} \cdot \pi \cdot \cancel{3} r^2 \cdot \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$-5 = 4\pi (2)^2 \frac{dr}{dt}$

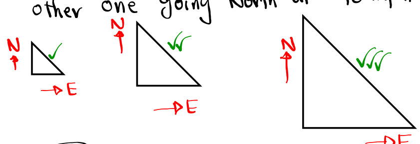
Radius is decreasing

$\frac{dr}{dt} = \frac{-5}{16\pi}$ in/hr

Two Cars leave an intersection of two Streets at the Same time.

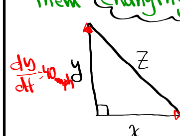
Assuming the Streets are ⊥ to each other.

One going east at 30 mph, and the other one going North at 40 mph.



How fast the distance between them changing after 1 hr

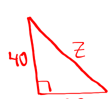
$\frac{dz}{dt} = ?$



$x^2 + y^2 = z^2$

$\frac{d}{dt}[x^2 + y^2] = \frac{d}{dt}[z^2]$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$



$z = 50$

$30^2 + 40^2 = z^2$

$x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$

$30 \cdot 30 + 40 \cdot 40 = 50\frac{dz}{dt}$

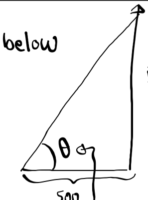
$900 + 1600 = 50\frac{dz}{dt}$

$2500 = 50\frac{dz}{dt}$

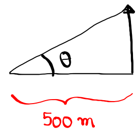
$\frac{dz}{dt} = 50$ mph

March 28, 2022

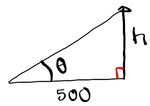
Consider the drawing below



Angle is changing.

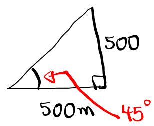


$\frac{d\theta}{dt}$ How fast is the angle changing after 5 seconds



$\tan\theta = \frac{h}{500}$

after 5 Seconds



$500\tan\theta = h$

$\frac{d}{dt}[500\tan\theta] = \frac{d}{dt}[h]$

$500 \cdot \sec^2\theta \cdot \frac{d\theta}{dt} = \frac{dh}{dt}$

$500 \cdot \sec^2 45^\circ \cdot \frac{d\theta}{dt} = 100$

$\cos 45^\circ = \frac{1}{\sqrt{2}}$

$\sec 45^\circ = \sqrt{2}$

$\overset{5}{\cancel{500}} \cdot [\sqrt{2}]^2 \frac{d\theta}{dt} = \cancel{100}^{1}$

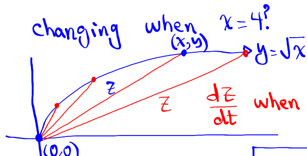
$5 \cdot 2 \frac{d\theta}{dt} = 1$

$\frac{d\theta}{dt} = \frac{1}{10}$ Rad/sec.

An object is moving along the curve $y = \sqrt{x}$ such that $\frac{dx}{dt} = 5$ in/min.

$y^2 = x$

How fast is its distance from the origin changing when $x = 4$?



$\frac{dz}{dt}$ when $x = 4$, $y = \sqrt{4} = 2$

Distance formula $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$z = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$ $\quad z = \sqrt{x^2 + x}$

$z^2 = x^2 + x$ $\quad \frac{d}{dt}[z^2] = \frac{d}{dt}[x^2 + x]$ $\quad z = \sqrt{4^2 + 4}$

$z = \sqrt{20}$

$2z\frac{dz}{dt} = (2x + 1)\frac{dx}{dt}$

$2 \cdot \sqrt{20}\,\frac{dz}{dt} = (2 \cdot 4 + 1) \cdot 5$

$4\sqrt{5}\,\frac{dz}{dt} = 45$

$\frac{dz}{dt} = \boxed{\frac{45}{4\sqrt{5}} \text{ in/min}}$

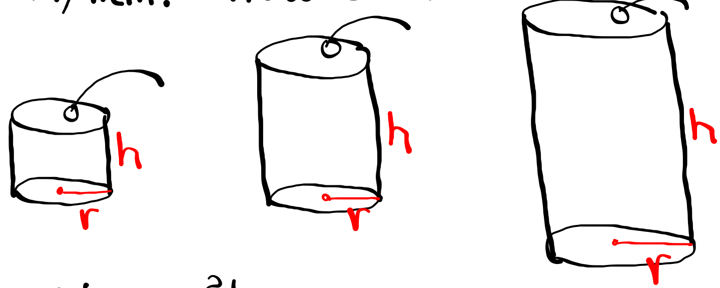
$\frac{45 \cdot \sqrt{5}}{4\sqrt{5}\sqrt{5}} = \frac{\overset{9}{\cancel{45}}\sqrt{5}}{4 \cdot \cancel{5}} = \frac{9\sqrt{5}}{4}$ in/min.

Next Week ⇒ Spring Break ⇒ No School

I will have my regular office hrs.

SG 12 ⇒ Extra Credit (25 points)

A cylindrical tank with radius 5 m is being filled with water at the rate of $3\ m^3/min$. How fast is the water level rising?



$$\frac{dV}{dt} = 3\ m^3/min.$$

$$V = \pi r^2 h$$

$$V = \pi \cdot 5^2 h$$

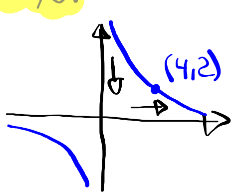
$$V = 25\pi h$$

$$\frac{dV}{dt} = 25\pi \frac{dh}{dt}$$

$$3 = 25\pi \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{3}{25\pi}\ m/min.$$

An object is moving along a hyperbola $xy=8$. As it is reaching the point $(4,2)$, the y-coordinate is decreasing at the rate of 3cm/s. How fast is the x-coordinate changing?



$xy=8$

$4(2)=8$

$8=8$ ✓

$$\frac{dy}{dt} = -3cm/s.$$

$$\frac{d}{dt}[xy] = \frac{d}{dt}[8]$$

$$\frac{dx}{dt} \cdot y + x \cdot \frac{dy}{dt} = 0$$

$$\frac{dx}{dt} \cdot 2 + 4 \cdot (-3) = 0$$

$$2\frac{dx}{dt} = 12$$

$$\boxed{\frac{dx}{dt} = 6cm/sec}$$

March 28, 2022

A Spherical Snowball is melting and its surface area decreases at rate of $1\ cm^2/min$. $\frac{dS}{dt} = -1$

Find the rate at which the diameter changes when the diameter is 10 cm.

$D = 2r$

$r = \frac{D}{2}$

Sphere $V = \frac{4}{3}\pi r^3$ $\quad S = 4\pi r^2$

$S = 4\pi \cdot \left(\frac{D}{2}\right)^2$

$S = \pi D^2$

$\frac{dS}{dt} = \pi \cdot 2D \cdot \frac{dD}{dt}$

$-1 = 2\pi \cdot (10) \frac{dD}{dt}$

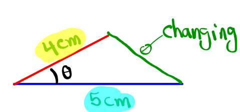
$\frac{dD}{dt} = \frac{-1}{20\pi}$ cm/min

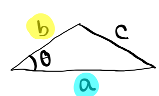
Diameter is decreasing.

Two sides of a triangle are 4m and 5m. the angle between them is increasing at the rate of .06 rad/s.

How fast is the third side changing with angle between known side is $\frac{\pi}{3}$.

$\frac{d\theta}{dt} = .06$ rad/s.





Law of Cosines:

$c^2 = a^2 + b^2 - 2ab\cos\theta$

$c^2 = 5^2 + 4^2 - 2\cdot5\cdot4\cos\theta$

$c^2 = 41 - 40\cos\theta$

when $\theta = \frac{\pi}{3}$

$c^2 = 41 - 40\cos\frac{\pi}{3}$

$c^2 = 41 - 40\cdot\frac{1}{2}$

$= 41 - 20$

$\boxed{c^2 = 21}$

$2c\frac{dc}{dt} = 0 - 40\cdot(-\sin\theta)\cdot\frac{d\theta}{dt}$

$\cancel{2}c\frac{dc}{dt} = \overset{20}{\cancel{40}}\sin\theta\frac{d\theta}{dt}$

$\sqrt{21}\frac{dc}{dt} = 20\cdot\sin\frac{\pi}{3}\cdot(.06) \rightarrow \boxed{\frac{dc}{dt} = ?}$